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Formulation of a point reactor kinetics model based on the neutron telegraph equation

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ABSTRACT

A Telegraph model of the Point Reactor Kinetics (TPRK) is developed based on the monoenergetic, non-fractional order telegraph approximation of the neutron transport equation. The model was used to develop a complete theory of the point reactor kinetics based on the non-fractional neutron telegraph equation. The inhour equation in the new form was obtained alongside different known reactor kinetics approximations applied to the model and their analytical solutions. The matrix form of the model was also obtained, which is needed for numerical solutions of the model. We solved this model for a step insertion of reactivity for an infinite thermal nuclear reactor system. In our model the value of the velocity of the neutrons propagation is finite and equals $v/\sqrt{3}$ because of the neutron telegraph equation. Although this value is still under research, it is different from infinite as in the diffusion approximation which contradicts causality. Values of the relaxation time (τ), which is the main difference between the (TPRK) and the Diffusion Point Reactor Kinetics (DPRK), is studied for different materials based on theoretical approach and its effect on the neutron density calculations have been calculated. The most important notice in the solution of the (TPRK) model is that there is a relaxation (retardation) in the time response of the solution. This is similar to what was found earlier in the literature concerning the solutions of the neutron telegraph equation.

1. Introduction

The Diffusion modeled Point-Reactor Kinetics (DPRK) equation is considered the primitive milestone of nuclear reactor kinetics. This is because it deals with a point reactor in which neutron density is only time dependent and with an equation modeled and built on the neutron diffusion equation which is considered as the correct first order approximation of the neutron transport equation.

Even though the neutron diffusion equation is simple to handle and solve, it has a very unfavorable outcome: That of the infinite propagation velocity of the neutrons. This, of course, contradicts causality principle. That the neutron diffusion equation or any parabolic differential equation (which the neutron diffusion equation is categorized under) has an infinite velocity can be found with detail in several fundamental nuclear reactor theory books. For e.g. Weinberg and Noderer (1951), Weinberg and Wigner (1958), Meghreblian and Holmes (1960) and Beckurts and Wirtz (1964).

Thus, the treatment of the neutron transport as a diffusion process has only limited validation. This can be deduced by considering the solution given by Beckurts and Wirtz (1964) to the time-dependent diffusion equation for a pulsed neutron experiment. For the simple problem of a point source located at the origin in an infinite medium, they obtained the following solution:

$$\phi(r,t) = \frac{Q}{\sqrt{v}(4\pi D)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4Dvt} - v\Sigma_a t\right) \tag{1}$$

where $\phi(r,t)$ is the neutron flux, v the neutron velocity, Σ_a the absorption cross section, D the diffusion coefficient, Q the source strength, and r is the distance between the neutron source and the point at which the flux is calculated.

From Eq. (1), it can be seen that for $t = 0$, the neutron flux vanishes everywhere except at the point $r = 0$; this result is expected since the source exists there. On the other hand, for incredibly very small time $t = \epsilon > 0$, $\phi(r,t)$ vanishes nowhere although it is small for all $r \neq 0$. In reality, however, $\phi(r,t)$ must vanish outside the sphere $r = v \times \epsilon$ because in the time ϵ no neutron can traverse a path longer than $v \times \epsilon$. The time-dependent diffusion equation does not describe this effect correctly.

The lack of the neutron diffusion equation can also be attributed to the fact that Fick's law, which the neutron diffusion equation is

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