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Analytical solution of the Telegraph Point Reactor Kinetics model during the cold start-up of a nuclear reactor

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ABSTRACT

We solve the new model of the Point Reactor Kinetics (PRK) equations developed based on the Telegraph approximation of the neutron transport equation analytically for a linear insertion of reactivity typically introduced by lifting the control rods discontinuously and manually during the cold start-up of a subcritical nuclear reactor. The Telegraph model introduces a new parameter called the Relaxation Time (τ) and we study its impact on the analytical solution for this case of reactivity insertion and for several speeds of lifting the control rods. We find that as the Relaxation time increases, the solution response is relaxed behind that of the diffusion model which was solved earlier in the literature for the same case. On the other hand, as the Relaxation time tends to zero, we find that the response of the neutron density tends to that of the diffusion case yielding a verification for the new proposed solution. Moreover, when reducing the Control Rod lifting speed, the effect of the relaxation time and hence the Telegraph approximation is reduced and it approaches that of the diffusion. We discuss Both mathematical and physical reasons for the cause of these behaviors.

1. Introduction

During cold startup of nuclear reactors, the response of the neutron density to the variations of several parameters is of utmost importance to ensure stable operation of the Reactor. These parameters include for e.g. speed and duration of control rods lifting, reactivity inserted and the external source strength. To formulate a relation showing the dependence of the neutron density inside a nuclear reactor on these different parameters, we need to understand how the nuclear reactor is cold started-up. In this paper, the startup scenario is similar to the one found in several papers, for e.g. (Zhang et al., 2008; Palma et al., 2009; Polo-Labarrios and Espinosa-Paredes, 2012). This startup scenario includes placing an external source of neutrons inside a sub-critical nuclear reactor, while the control rods of the reactor are manually lifted discontinuously in time so that positive reactivity is inserted into the system to start it up. In this case, the average temperature of the reactor core is still low and the power added is small, so that the temperature feedback effect is neglected (Zhang et al., 2008; Palma et al., 2009; Polo-Labarrios and Espinosa-Paredes, 2012; Da Silva and Narain, 2012; Nahla, 2017).

Zhang et al. (2008) and Palma et al. (2009) solved analytically, albeit with different approaches, the problem using the well-known Diffusion Point Reactor Kinetics (DPRK) model considering one delayed neutrons group:

$$\left.\begin{aligned} \frac{dn(t)}{dt} &= \frac{\rho(t)-\beta}{\Lambda}n(t) + \lambda C(t) + q \\ \frac{dC(t)}{dt} &= \frac{\beta}{\Lambda}n(t) - \lambda C(t) \end{aligned}\right\} \tag{1}$$

where $n(t)$ is the neutron density with temporal dependence, $\rho(t)$ is the reactivity, β is the total delayed neutrons fraction of the fission neutrons, Λ is the prompt neutrons mean generation time, q is the external neutron source density, λ is the average delayed neutrons decay constant for the delayed neutrons groups, and $C(t)$ is the delayed neutrons precursor concentration.

Furthermore, they considered a reactivity insertion that is variable in time, given by:

$$\rho(t) = \begin{cases} \rho_s + rt & 0 \leqslant t < t_0 \\ \rho_s + rt_0 & t \geqslant t_0 \end{cases} \tag{2}$$

where ρ_s is the sub-critical reactivity; r is the inserted reactivity velocity or the velocity of the control rods; t is the time; t_0 is the

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