Reactor Dynamics Based on the Neutron Telegraph Equation

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INTRODUCTION

We solve the neutron Telegraph Point Kinetics Model developed in [1-3] for different Reactor Transients with Temperature feedback including a Temperature Dependent Prompt Feedback Coefficient $\gamma(T)$. We also compare the new model to that of the Traditional Diffusion Point Reactor Kinetics model. The newly suggested 8 group of delayed neutrons reported in OECD/NEA's WPEC–6 report [4] is used instead of the traditional 6 groups. The new model introduces a new parameter that affect the solution, called the Relaxation Time τ, which the traditional diffusion model neglects. We discuss the mathematical and physical reasons resulting in the differences shown between both models when treating Reactor Dynamics for the different Reactor Transients reported and discuss the significance of the research.

DESCRIPTION OF THE ACTUAL WORK

In this research, we solve the newly developed Point Reactor Telegraph model [1-3] for several Reactor Transients with Prompt Feedback. We study the Reactor dynamics using the newly developed model for different reactor transients with varying parameters that include the Reactor Geometrical Buckling B^2, the Rate of Reactivity Insertion a, and the Prompt Feedback Coefficient γ. We also compare our results to the traditional Diffusion Point Reactor model to show the differences between both models as well as the area of applicability for both models.

In recent publications [1-3], we developed the following Telegraph model, cf. equation (2), which is based on the full form of the P_1 approximation to the neutron transport equation, in contrast to the diffusion approximation in which the time derivative term of the neutron current $\mathbf{J}(\vec{\mathrm{r}},\mathrm{t})$, viz. $(\partial\mathbf{J}(\vec{\mathrm{r}},\mathrm{t})/\partial\mathrm{t})$, is neglected from the P_1 approximation system of equations. Keeping this term results in the addition of a new parameter that affects the solution, called the Relaxation time (τ), and is given by [1-3]:

$$\tau = \frac{3D}{\mathrm{v}} \tag{1}$$

Here D is the diffusion coefficient and v is the neutron speed. We can see from Equation (1) that the values of τ depends on the neutron speed v, as well as on the diffusion coefficient D and hence Σ_{tr} of the medium materials, since $D = 1/3\Sigma_{tr}$, where Σ_{tr} is the neutrons Transport macroscopic cross section and hence, τ depends on the medium material and its atomic density. τ theoretical calculated values are given in Table 1 for various materials, for a $2200\ m/sec$ neutrons.

TABLE I. Calculated Values of Relaxation Time (τ) for Relevant Materials

Material	D (cm)	τ (sec)
Light water H_2O	0.143	1.95×10^{-6}
Heavy water D_2O	0.84	1.1455×10^{-5}
Helium He	19046	2.6×10^{-1}
Beryllium Be	0.42	6×10^{-6}
Beryllium oxide BeO	0.71	9.7×10^{-6}
Graphite C	0.92	1.2546×10^{-5}
Uranium oxide UO_2	0.62	8.456×10^{-6}

As we can see from Table 1, the values of τ is between 10^{-1} sec, for e.g. Gaseous mediums, to 10^{-6} sec. We had shown that [1-3] when $\tau \to 0$, the new telegraph model solution tends to that of the diffusion which is actually the $\tau = 0$ case. Hence, we can project that for small τ values, the difference between the Telegraph model solution and the diffusion model solution is small. We note that for the same neutron energy, we can represent the relaxation time τ of a homogenous mixture as the sum of all individual relaxation times τs of the materials that form the mixture, since the diffusion coefficients are additive for a homogenous mixture [2]. The values range of τ used in this research is from 10^{-1} sec to 10^{-4} sec since according to [2], 10^{-4} sec value for τ is calculated using parameters of a typical PWR.

The Telegraph Point Reactor Kinetics model is given by [2]:

$$\left.\begin{array}{c} \tau_0 \dfrac{d^2p(t)}{dt^2} - \tau_0\left[\dfrac{\rho(t)-\mu}{\Lambda}\right]\dfrac{dp(t)}{dt} + \dfrac{dp(t)}{dt} = \dfrac{\rho(t)-\beta}{\Lambda}p(t) + \\ \displaystyle\sum_{i=1}^{m}\lambda_i C_i(t) + \tau_0\sum_{i=1}^{m}\lambda_i\frac{dC_i(t)}{dt} + q(t) + \tau_0\frac{\partial q(t)}{\partial t} \\ \dfrac{dC_i(t)}{dt} = \dfrac{\beta_i}{\Lambda}p(t) - \lambda_i C_i(t) \end{array}\right\} \tag{2}$$

Here τ is the relaxation time of a finite medium while τ_0 is the relaxation time of an infinite medium, $p(t)$ is the Reactor power with temporal dependence, $\rho(t)$ is the reactivity, β is the total delayed neutrons fraction of the fission neutrons while β_i is the delayed neutron fraction of the fission neutrons for the i^{th} delayed neutrons precursor group, Λ is the prompt neutrons mean generation time, q is the external neutron source density, m is the number of the delayed neutron precursor groups, λ_i is the delayed neutron decay constant for the i^{th}delayed neutrons precursor