Annals of Nuclear Energy 177 (2022) 109277

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

# Extending a low-order inhomogeneous adjoint equations model to a higher-order model with verification on integral applications

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ARTICLE INFO

*Article history:*
Received 27 November 2021
Received in revised form 5 May 2022
Accepted 10 June 2022
Available online 1 July 2022

*Keywords:*
Nodal Expansion Method (NEM)
Adjoint
Lagrange Multiplier
Axial Offset (AO)
Perturbation analysis
Sensitivity analysis
IAEA

ABSTRACT

A higher-order nodal mathematical inhomogeneous adjoint model conjugate to the NEM-M2B2 nodal diffusion forward model is developed and introduced in this research. Verification of the developed model is presented through applications in perturbation analysis and the IAEA-3D benchmark including adjusted forms of it. This paper's objective is to explore ways of extending and optimizing a mathematical adjoint capability suitable for use in an industrial reactor code, such that it becomes not merely an approximate but rather the exact adjoint counterpart to the typically used higher-order nodal forward solvers used in mature industrial reactor codes. Specifically, it is investigated how to upgrade an already available lower-order nodal mathematical adjoint solver towards higher-order accuracy. An example of the latter is the lower-order nodal adjoint solver used in the ARTEMIS reactor code, in the technical context of stabilization and acceleration of embedded control rod search mechanisms. Though the latter adjoint solver proved suitable for the needed preconditioning purposes, while also enabling the benefit of computationally very lean adjoint iterations, several future developments could benefit from having a higher-order adjoint nodal solver available as well. By using a preconditioned form of the base NEM-M2B2 nodal diffusion forward model and by using variational analysis, we have obtained a higher-order nodal mathematical adjoint that can have a physical interpretation associated with it as a Lagrangian multiplier. The nodal mathematical adjoint is then developed for the Axial Offset (AO) as a Response of Interest (RoI) which leads to an inhomogeneous adjoint system of equations. A solution verification of the adjoint developed is done through analyzing the effects coming from perturbations in the absorption and the scattering cross-sections. The applications investigated include axially and radially traveling perturbations along the reactor's core. Several locations for the traveling perturbations are chosen to represent important locations in the core. Comparison between the low-order and the higher-order adjoint models is conducted. The forward model is set to the NEM-M2B2 nodal diffusion equation for both adjoints during the comparison. The higher-order adjoint model developed show consistent results in comparison to its lower-order sibling, suggesting the preference of using the developed higher-order model for adjoint computations.



## 1. Introduction

Interests have emerged in exploring technical-scientific ways of upgrading an already available lower-order nodal adjoint solution capability, such as available in the ARTEMIS reactor code, for stabilization and acceleration of embedded control rod search mechanisms (van Geemert, 2020), towards higher-order nodal accuracy. A recent capability in ARTEMIS related to code acceleration is the Generalized Perturbation Theory (GPT) (Geemert, 2014). The GPT capability implemented in ARTEMIS enables the goal-oriented numerical solution of inhomogeneous adjoint equations based on a 3D adjoint/forward multi-group nodal diffusion formulation derived based on a low-order (quadratic polynomial based) simplified Nodal Expansion Method (NEM)-M0 approach. It has become possible through this GPT module to compute approximated but nonetheless adequate 3D multi-group sensitivity fields connected to several different kinds of Responses of Interest (RoIs) (e.g., $\lambda$-eigenvalue (related directly to the core reactivity $\rho$ through $\rho = 1 - \lambda$) Geemert (2014), and Axial Offsets (AOs) (Geemert, 2020)).

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https://doi.org/10.1016/j.anucene.2022.109277
0306-4549/